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Towards understanding the robustness of energy distribution networks based on macroscopic and microscopic evaluations

Jiming Liu*, Benyun Shi

Department of Computer Science, Hong Kong Baptist University, Hong Kong

HIGHLIGHTS

- ► This paper evaluates distribution robustness by defining a notion of network entropy.
- ► The disruption impacts on individual node are evaluated by a failure spreading model.
- ► The robustness of the U.S. natural gas distribution network is studied.
- ► Results reveal pipeline bottlenecks, the node rank, and potential storage locations.
- ► Possible strategies for mitigating the impacts of supply disruptions are discussed.

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ABSTRACT

Supply disruptions on one node of a distribution network may spread to other nodes, and potentially bring various social and economic impacts. To understand the performance of a distribution network in the face of supply disruptions, it would be helpful for policy makers to quantitatively evaluate the robustness of the network, i.e., its ability of maintaining a supply–demand balance on individual nodes. In this paper, we first define a notion of network entropy to macroscopically characterize distribution robustness with respect to the dynamics of energy flows. Further, we look into how microscopic evaluation based on a failure spreading model helps us determine the extent to which disruptions on one node may affect the others. We take the natural gas distribution network in the USA as an example to demonstrate the introduced concepts and methods. Specifically, the proposed macroscopic and microscopic evaluations provide us a means of precisely identifying transmission bottlenecks in the U.S. interstate pipeline network, ranking the effects of supply disruptions on individual nodes, and planning geographically advantageous locations for natural gas storage. These findings can offer policy makers, planners, and network managers with further insights into emergency planning as well as possible design improvement.

1. Introduction

Energy distribution networks, such as natural gas transmission networks (Kabirian and Hemmati, 2007), electricity networks (Shaw et al., 2009, 2010), and integrated energy networks (Jamasb and Pollitt, 2008), play essential roles in delivering energy resources from suppliers to meet the demands of consumers through intermediate nodes. Historical data shows that energy supply as well as distribution infrastructures can be severely disrupted by serious accidents caused by either natural or man-made hazards (Burgherr and Hirschberg, 2008). Such disruptions may bring various degrees of economic and social impacts (Sovacool, 2008), such as the impacts of energy shocks on the financial market (Scholtens and Boersen, 2011). Furthermore, due to the interconnection between energy suppliers and consumers through distribution infrastructures, certain supply disruptions on one node in a distribution network may spread to other nodes of the network (van der Vleuten and Lagendijk, 2010a,b). For example, in 2005, hurricanes Katrina and Rita damaged gas production and transmission in the Gulf of Mexico, which caused further supply shortages along the East Coast of the USA. One major reason is that consumers on the East Coast could no longer obtain their natural gas from other suppliers (e.g., Texas) due to the limitation of pipeline capacity (Federal Energy Regulatory Commission, 2006). To understand the performance of an energy distribution network under potential supply disruptions, it would be informative and useful to formally evaluate the robustness of the distribution network as a whole.

* Corresponding author. Tel.: +852 34117088; fax: +852 34117892.
E-mail address: jiming@comp.hkbu.edu.hk (J. Liu).

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Existing studies on complex networks have examined two types of robustness, i.e., static robustness and dynamic robustness (Boccaletti et al., 2006). Static robustness addresses the ability of a network to maintain its structural connectivity (e.g., the average shortest path length of the network) after a series of random failures or malicious attacks. Dynamic robustness examines the effects of cascading failures on the structural connectivity of a network by considering the redistribution of commodity flows on the network (e.g., electricity on power grids), where the structural failures may be highly affected by the distribution capacity of the network (Motter and Lai, 2002). Since the major function of an energy distribution network is to meet the demand of each consumer, we are specifically interested in how potential supply disruptions affect the supply–demand balance on individual nodes through the dynamics of energy flows on the network. In this case, the robustness of a distribution network in this paper reflects its ability to maintain a supply–demand balance on individual nodes in the face of supply disruptions. From the perspective of complex networks (Albert and Barabási, 2002; Newman, 2003), the supply–demand balance on each individual node is determined by not only how the nodes connect with each other (i.e., network connectivity) but also how the energy flows on the network. Therefore, to evaluate the robustness of a distribution network, both the network connectivity and the energy flow dynamics on the network should be taken into consideration.

In this paper, we first examine how to macroscopically characterize distribution robustness with respect to the energy flow dynamics on a given distribution network. In the literature of complex networks, network entropy has been introduced to measure the robustness of structural connectivity of complex networks, where only the information of network connectivity (e.g., degree distribution) is concerned. Here, to characterize the robustness of energy flow dynamics, we formally define network entropy by modeling energy flows as random walks on the network. Such a definition reflects the pathway diversity of energy flows from suppliers to consumers. In other words, it measures the possibility of each unit of energy flows passing through the distribution paths of the network. Based on the Ruelle–Bowens random walk theory, our definition can serve as a metric for globally measuring distribution robustness. Accordingly, we can identify the bottlenecks of real-world distribution infrastructures by comparing their distribution capacities with the theoretically robust energy flows, which can be calculated by solving a network entropy optimization problem under certain distribution constraints. Next, to gain a deeper understanding of the disruption impacts on individual nodes, we look into how a microscopic evaluation based on a failure spreading model can help reveal the extent to which supply disruptions on one node may affect other nodes. Failure spreading models have been extensively adopted to model cascading failures in various complex networks such as power grids, economic and banking systems. In this paper, by defining the "failure" of an individual node as the imbalance of its supply and demand, we can evaluate the effects of supply disruptions on each of the individual nodes, and explore the interdependency among them. Furthermore, such a microscopic evaluation also provides a way to understand the definition of network entropy at the macroscopic level.

By using the publically available statistical data, we demonstrate our proposed concepts and methods in the case of the U.S. natural gas distribution network. Specifically, the macroscopic and microscopic evaluations introduced allow us to (i) precisely identify the transmission bottlenecks in the U.S. interstate pipeline network from the viewpoint of distribution robustness; (ii) quantitatively rank the effect of each state in terms of both the number of failures it may incur and the number of states that may cause its failure; and (iii) strategically plan advantageous storage locations that are geographically convenient for mitigating the effect of disruptions on other states. Based on our findings, we arrive at some implications for mitigating the impacts of supply disruptions on distribution networks. First, the existing work on complex systems approaches to planning in energy networks emphasizes the capability of autonomously making decisions by energy suppliers and consumers within the network (Beck et al., 2008). Along this line, the macroscopic evaluation of distribution robustness can provide a new way for policy makers to motivate all participants towards robust energy distribution. Second, together with the microscopic evaluation, our findings can offer some insights into (i) management of energy flows and transmission capacities, (ii) protection of critical nodes from being disrupted, and (iii) planning the location of energy storage from a systematic perspective.

The reminder of this paper is organized as follows. In the following section, we survey the related work and theoretical background about network robustness analysis. In Section 3, we describe in detail the notion of network entropy and the failure spreading model, which evaluate distribution robustness from macroscopic and microscopic viewpoints, respectively. We present a case study on the U.S. natural gas distribution network to demonstrate our proposed concepts and methods in Section 4, and discuss implications for mitigating the impacts of supply disruptions based on our findings in Section 4.5. Finally, we provide our conclusions in Section 5.

2. Theoretical background

Several areas of study are related to this work: (i) the characterization of network robustness based on the concept of network entropy, (ii) the use of failure spreading models to simulate the impacts of disruptions, (iii) the ranking of node importance according to various centrality measures, and (iv) the identification of transmission congestion using the concept of locational marginal price.

Many studies have shown that network entropy is positively correlated with network robustness, and can therefore serve as a metric for network robustness. For example, Wang et al. (2006) propose a definition of network entropy based on the degree distribution of a network, where the distribution function $P(k)$ represents the probability that a randomly selected node has exactly k neighboring nodes in the network. Similarly, other structural information, such as remaining degree distribution (Solé and Valverde, 2004) and community structure (Anand and Bianconi, 2009), has also been adopted to define network entropy. Recently, the Ruelle–Bowens random walk, which is a physics-inspired and mathematically profound theory, has received some attention in the study of complex networks. The relationship between the Ruelle–Bowens random walk and the structural robustness of a complex network has been explored by Demetrius and Manke (2005). According to the ergodic theory, Gómez-Gardeñes and Latora (2008) propose entropy rate to present the minimal amount of information necessary to describe a diffusion process in a network, where node degrees are used as a local structural information by random walkers. Delvenne and Libert (2011) further propose a node centrality measure (i.e., the relative importance of a node in a network) for complex networks based on the concept of entropy rate. However, most of the existing studies focus only on the robustness of structural connectivity of complex networks, few of them can be directly adopted to characterize the robustness of energy flow dynamics on distribution networks, i.e., the ability to maintain a supply–demand balance on individual nodes. In this paper, by treating energy flow dynamics as a Ruelle–Bowens random walk on a distribution network, we focus on defining network entropy with respect to the dynamics of energy flows to evaluate the robustness of the network at the macroscopic level.

Simulating the cascading failures in complex networks has drawn extensive attention in recent years. For example, Albert et al. (2004) and Kinney et al. (2005) have successively analyzed the effects of cascading failures in the North American power grid; Garas et al. (2010) and Lee et al. (2011) have independently studied the spreading of economic crisis on the global macroeconomic network using different cascading models; May and Arinaminpathy (2010) and Haldane and May (2011) have analyzed the risk spreading in banking systems based on a shock propagation model. Such a microscopic evaluation may help us to gain a deep understanding about how the structural connectivity of a network affects the dynamics on the network under specific disruptions. Studies have shown that both the structural connectivity of the network and the dynamics on the network determine the spreading procedure. Through simulation, the roles of each individual node during the spread of system failures, crisis, or risk can be precisely identified. With respect to a distribution network, in this paper, we adopt a failure spreading model to evaluate the extent to which supply disruptions on a node may affect the supply–demand balance of other nodes on the network. Moreover, based on the quantity of supply and demand on each node, we can further rank the importance and vulnerability of individual nodes, and explore the interdependency among the nodes in terms of failures caused by supply disruptions.

Various centrality measures, such as closeness centrality (i.e., measuring the average distance of a node to all other nodes) and betweenness centrality (i.e., quantifying the frequency of a node that occurs on a randomly chosen shortest path between two randomly chosen nodes), have been proposed in terms of network connectivity to investigate how important a node is in real-world systems. For example, Google and other web search engines assign a PageRank score to each page of the web according to how pages are connected to one another (Brin and Page, 1998), Blöchl et al. (2011) reveal the structure of modern economics by exploring the random-walk centrality and counting the betweenness of each sector of an economy. Meanwhile, extensive studies have investigated how species' importance may affect secondary extinctions in food webs through the use of dominator trees (Allesina and Bodini, 2004) and eigenvector-based measures (Allesina and Pascual, 2009). By adopting a failure spreading model in this paper, we can achieve a better understanding about the importance and vulnerability of each individual node on a distribution network.

In recent years, many studies have been focused on the network optimization problems, for example, optimizing distribution costs of an integrated energy network (Quelhas et al., 2007; Quelhas and McCalley, 2007), controlling package flows in communication networks (Shakkottai and Srikant, 2007), and identifying the congestion and valuation of natural gas transmission infrastructure (Lochner, 2011). To optimize distribution costs under the constraints of distribution capacities, most of them adopt the concept of locational marginal price (LMP). LMP was first introduced by Schweppe et al. (1988), and was further developed by Hogan (1992). The price equals the marginal cost of supplying one additional unit of energy at a location, which is generally calculated by solving a mathematical optimization problem. Different from those studies about optimizing distribution costs, in this paper, we propose a network optimization problem from the perspective of distribution robustness. We calculate the optimal energy flows on a distribution network by solving a constrained network optimization problem with respect to network entropy. By comparing the optimal quantity of energy flows with the real-world capacities of distribution infrastructures, we can further identify the bottlenecks in the network for robust distribution.

3. Proposed methods

3.1. Distribution networks and network entropy

Distribution networks for different energy resources may require different energy distribution infrastructures and technologies. For example, a natural gas distribution network consists of infrastructures such as compressor stations, metering stations, valves, and control stations. Since we aim to understand the robustness of distribution networks at the supply–demand level, the detailed modeling of specific distribution infrastructures may not help to improve our findings. In this paper, we model a distribution system as a network $G(V,E,W)$. Node set V of the network consists of N nodes, each of which represents either an energy supplier, or consumer, or an intermediary. An intermediary node may also consume some of the incoming flows and forward the rest to other nodes. The adjacent matrix $E=\{e_{ij}|e_{ij}\in\{0,1\}\}_{N\times N}$ of the network represents whether or not there is energy flows from v_i to v_j, which reflects the structural connectivity of the distribution network. Accordingly, the weighted matrix $W=\{w_{ij}\}_{N\times N}$ represents the quantity of energy flows between each pair of nodes, which reflects the dynamics of energy flows on the network.

Although Demetrius and Manke (2005) have proposed a definition of network entropy based on the Ruelle–Bowens random walk theory, they focus mainly on evaluating the static robustness of undirected networks. According to the theory, to macroscopically evaluate the robustness of energy flow dynamics on $G(V,E,W)$, one major problem is that most energy distribution networks are directed and acyclic, i.e., the energy cannot flow back to where it is produced. Similar to the method proposed by Allesina and Pascual (2009) on food webs, we attach to a distribution network a special node (i.e., a "root"), which provides the same quantity of production to all suppliers, and at the same time absorbs the same amount of consumption by consumers. As shown in Fig. 1, directed edges are added from the root node to v_1 and v_2, which are energy suppliers of the original distribution network. Meanwhile, the dashed lines pointing to the root node represent the consumptions of the nodes. With such a modification any distribution network becomes cyclic and we can further analyze its robustness based on the Ruelle–Bowens random walk theory. In a newly formed cyclic distribution network, a node is said to reach a supply–demand balance if for each node in the

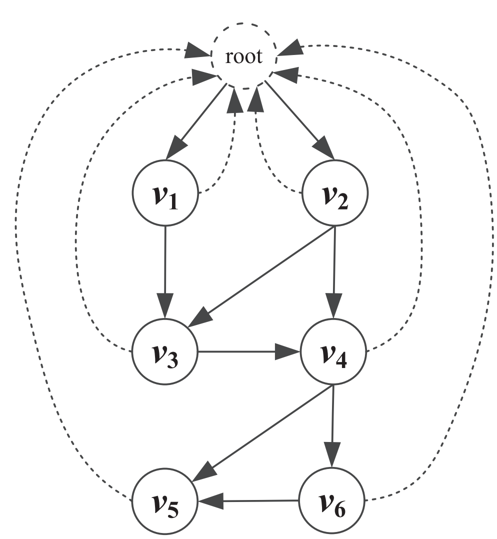

Fig. 1. An illustration of a distribution network with a special node (the "root" node). Solid lines represent real energy flows, while dashed lines pointing to the root node represent the consumptions of the nodes.

newly formed distribution network, the total energy inflows should equal to the total outflows. This property is also called *flow conservation* (Kumar et al., 2009).

We treat energy flows in the newly formed distribution network as the Ruelle–Bowens random walk P, in which $p_{ij} \in P$ represents the probability that a unit of flows which enters v_i is distributed along edge e_{ij} to v_j. Along this line, we have $p_{ij} = w_{ij}/\sum_k w_{ik}$. We denote the stationary state π_i as the expected quantity of inflows of v_i. In this case, the stationary state of v_i can be calculated as follows:

$$\pi_i = \sum_{e_{ki}=1} \pi_k \cdot p_{ki} \tag{1}$$

The nodes' stationary states are clearly interdependent; that is, the value π_i depends on the stationary states of its preceding nodes, $\{v_k | e_{ki} = 1\}$. Moreover, the stationary state distribution Π also depends on the stochastic process P. Here, we assume the stationary state of the root node to be unit one. Starting from the root node, the stationary state of each supplier $v_i \in S$ is determined by the quantity of its supply s_i, i.e., $\pi_i = s_i/\sum_{v_k \in S} s_k$. Consequently, the stationary states of other nodes can be calculated based on Eq. (1). The entropy for mutually dependent distributions Π and P is given by

$$H(\Pi * P) = H(\Pi^s) + H(P, \Pi^{-s} | \Pi^s) \tag{2}$$

where $H(\Pi^s)$ is the entropy of the stationary distribution for all independent supplier nodes $v_i \in S$ (e.g., v_1 and v_2 in Fig. 1), $H(P, \Pi^{-s} | \Pi^s)$ is the entropy of the random walk P condition upon Π^s, where Π^{-s} is the stationary distribution of the other nodes $v_i \in V \backslash S$. Based on the Ruelle–Bowens random walk theory, the network entropy can be calculated as follows:

$$H(\Pi^s) = -\sum_{v_i \in S} \pi_i \log \pi_i \tag{3}$$

and

$$H(P, \Pi^{-s} | \Pi^s) = -\sum_{v_i \notin S, e_{ij}=1} \pi_i p_{ij} \log p_{ij} \tag{4}$$

Such a definition of network entropy reflects the long-term behavior of the energy flow dynamics; in other words, it measures the pathway diversity of energy flows on a distribution network. The larger the network entropy, the less predictable the random walk; that is, the more equally the flows are carried on all possible distribution paths. Such a macroscopic characterization sheds light on the relationship between distribution robustness and energy flow dynamics in a distribution network, which can help to enhance the robustness of the network by maximizing the network entropy under certain distribution constraints. In Section 4.2, we will demonstrate our method in the case of the U.S. natural gas distribution network to theoretically calculate the robust natural gas flows on the network.

3.2. The failure spreading model

Although network entropy can macroscopically characterize distribution robustness in terms of flow dynamics, it cannot be used to measure the extent to which supply disruptions on one node may affect the other nodes in the network. To achieve this, in the following, we introduce our failure spreading model in terms of an energy distribution network. We assume that a supply disruption on a node $v_i \in V$ of $G(V,E,W)$ may cause its expected total inflows equal to zero, i.e., $\pi_i = 0$. As a result, the stationary states of the downstream nodes that can be reached through the network by v_i will be affected. Once a node is affected, it will first satisfy its own demand, which is represented by the dashed line to the "root" node. Then, the rest of the energy will be distributed to its downstream neighbors (i.e., along the solid lines), where the quantity along each downstream edge is determined proportional to their original weight. Consequently, a new weight matrix $\hat{W}^i$ is formed, which reflects the energy flows on the network after the supply disruption on v_i. We use an impact matrix $A^i = W^i - \hat{W}^i$ to represent the difference of the two weight matrixes, where the element a_{jk} of A^i represents the extent to which disruptions to v_i indirectly affect v_k through v_j.

According to the failure spreading model, nodes may differ in their ability to afford disruptions. For example, an node that receives energy resources from multiple suppliers is more likely to afford disruptions on its upstream nodes. A node fails to maintain its supply–demand balance when the total inflows of the node are less than its own demand. Based on this criteria, each node v_i is associated with a set of dominant nodes D_i, whose disruption may cause its failure. By doing so, we can easily identify which nodes are important (i.e., the nodes whose disruption may cause extensive failures) and vulnerable (i.e., the nodes with a large dominant set). Further, to examine the interdependency among the nodes from a systemic viewpoint, we adopt a hierarchical structure to represent the relationships between the nodes' dominant sets. The structure is formed as follows: for each node v_j, if v_i's dominant set D_i is the minimal superset of D_j, then a directed edge will be formed between v_i to v_j. By doing so, each path in the hierarchical structure must be dominated by at least one node. For example, for path $v_1 \to v_2 \to v_3$, we have $D_1 \supset D_2 \supset D_3$. However, for two nodes (e.g., v_i and v_j) on different paths, their dominant sets satisfy $D_i \backslash D_j \neq \emptyset$ and $D_j \backslash D_i \neq \emptyset$, which means that they may be dominated by different nodes. Such a hierarchical structure can help us plan systematic mitigation strategies for potential disruptions, such as identifying advantageous natural gas storage locations in Section 4.4.

3.3. Understanding network entropy from a microscopic viewpoint

In addition to exploring the robustness properties of each individual node, the results of the failure spreading model can also help quantify the robustness of a distribution network defined by network entropy. Given a distribution network $G(V,E,W)$, supply disruptions on a node v_i may result in an impact matrices, A^i. Denote I_i the set of nodes that are affected by a disruption on v_i. For each node $v_k \in I_i$, if v_k has more than one downstream neighbor, the energy flows will be split and the effect of the disruption will be mitigated. The mitigation capacity of v_k can be measured by the impacts on its downstream neighbors:

$$MC^i(k) = -\sum_{j, a_{kj} \neq 0} \frac{a_{kj}}{A_k^i} \log \left(\frac{a_{kj}}{A_k^i} \right) \tag{5}$$

where $v_k \in I_i$, and $A_k^i = \sum_j a_{kj}$ is the total impacts of v_k on its downstream neighbors. Moreover, the mitigation capacity of the network with respect to supply disruption on v_i can be calculated by $MC^i = \sum_{k \in I_i} A_k^i \cdot MC(k)$. To quantify the robustness of the network for arbitrary disruptions, we assume that each supplier node may be disrupted with equal probability. Then, the distribution robustness can be measured by the weighted average value of mitigation capacities with respect to all possible disruptions:

$$\Gamma = \frac{\sum_{v_i \in S} MC^i}{\sum_{v_i \in S} \pi_i} \tag{6}$$

Essentially, such a quantification at the microscopic level is equivalent to network entropy defined at the macroscopic level. Based on the definition of an impact matrix, for each node v_k we have $a_{kj} = A_k^i p_{kj}$, where $p_{kj} = w_{kj}/\sum_i w_{ki}$. Therefore, $MC^i(k) = -\sum_j p_{kj}$

Table 1
The geographical information on the U.S. natural gas distribution network.

Region/ country	Division	ID	Nodes
Northeast	New England	R1D1	Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut
Northeast	Mid-Atlantic	R1D2	New York, Pennsylvania, New Jersey
Midwest	East North Central	R2D3	Wisconsin, Michigan, Illinois, Indiana, Ohio
Midwest	West North Central	R2D4	Missouri, North Dakota, South Dakota, Nebraska, Kansas, Minnesota, Iowa
South	South Atlantic	R3D5	Delaware, Maryland, District of Columbia, Virginia, West Virginia, North Carolina, South Carolina, Georgia, Florida
South	East South Central	R3D6	Kentucky, Tennessee, Mississippi, Alabama
South	West South Central	R3D7	Oklahoma, Texas, Arkansas, Louisiana
West	Mountain	R4D8	Idaho, Montana, Wyoming, Nevada, Utah, Colorado, Arizona, New Mexico
West	Pacific	R4D9	Alaska, Washington, Oregon, California, Hawaii
Canada	–	CANA	Alberta, British Columbia, Saskatchewan, Ontario, Manitoba, New Brunswick, Quebec
Foreign	–	–	Mexico, Japan, Russia, Egypt, Equatorial Guinea, Nigeria, Algeria, Australia, Trinidad and Tobago, Malaysia, Oman, Qatar, Norway, United Arab Emirates

$\log(p_{kj})$ and $MC^i = -\sum_{k \in I_i}(A_k^i \cdot \sum_j p_{kj} \log(p_{kj}))$. When a supplier v_i is disrupted, A_i^i becomes π_i (i.e., the total supply of v_i). For each $v_k \in I_i$, $A_k^i = \sum_{\{j_1 \cdots j_m\} \in \Sigma_{ik}}(\pi_i \cdot p_{ij_1} \cdots p_{j_{m-1}j_m} \cdot p_{j_m k})$, where Σ_{ik} is the set of paths from v_i to v_k. Together with the flow conservation on each node, $A_k^i = \sum_{e_{jk}=1} A_j^i \cdot p_{jk}$, we have $\sum_i A_k^i = \pi_k$. Therefore, Eq. (6) becomes $\Gamma = H(P, \Pi^c | \Pi^s)/\sum_{v_i \in S}\pi_i$, where $\sum_{v_i \in S}\pi_i$ is constant when energy supply is known in advance.

4. Case study: analyzing the robustness of the U.S. natural gas distribution network

In this section, to demonstrate the introduced concepts and methods, we take the natural gas distribution network in the USA as an example to examine its distribution robustness. After introducing the distribution network in Section 4.1, we show how the bottlenecks of the U.S. interstate pipelines can be identified using our macroscopic evaluation (i.e., network entropy) in Section 4.2. Next, based on our microscopic failure spreading model, we rank the importance and vulnerability of the nodes in Section 4.3, and identify geographically advantageous storage locations that are potential to mitigate the impacts of supply disruptions in Section 4.4. Finally, we discuss several mitigation strategies based on our findings in Section 4.5.

4.1. The U.S. natural gas distribution network

Based on the publically available statistical data, we model the United State natural gas distribution network with 73 nodes, consisting of 50 U.S. states, the District of Columbia, the Gulf of Mexico, seven Canadian provinces (i.e., Alberta, British Columbia, Saskatchewan, Ontario, Manitoba, New Brunswick, Quebec), and 14 foreign countries. The geographical information[1] for the nodes is shown in Table 1, which may help identify geographically advantageous natural gas storage locations based on our findings.

The movement of natural gas among the 73 nodes forms a directed weighted network. The data are obtained from the U.S. Energy Information Administration (EIA, 2007b). Based on the data set, we use the quantity of natural gas movement from one node v_i to another v_j, measured by million cubic feet per day (i.e., *MMcf/d*), as the weight of the edge e_{ij}, that is, w_{ij}, to reflect their interdependency. Fig. 2 illustrates the schematic model of the

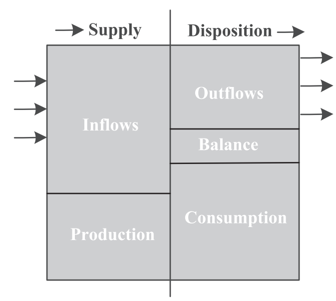

Fig. 2. The schematic model of a node in the U.S. natural gas distribution pipeline network.

node in the U.S. natural gas distribution network based on annual statistical data from EIA (2007d), in which inflows consist of interstate receipts and receipts across U.S. borders, whereas outflows include interstate deliveries and deliveries across U.S. borders. The flows injected into (withdrawn from) underground storage are considered as self-consumption (self-production). The balancing item, which represents the difference between the sum of the components of natural gas supply and the sum of the components of natural gas disposition, allows the input–output table to reach a balance.

4.2. Identifying bottlenecks in the U.S. natural gas pipeline network

The U.S. natural gas pipeline network constitutes the major transmission infrastructure for natural gas distribution among the U.S. states. Each pipeline is associated with a physical constraint, i.e., the transmission capacity (measured by *MMcf/d*), which regulates the maximal amount that can flow through the pipeline within a day (EIA, 2007e). From the perspective of distribution robustness, it is essential to identify bottlenecks in the U.S. pipeline network so that the transmission capacity can be managed appropriately. To achieve this, we first calculate the robust energy flows (*RCF*) based on the statistics about the U.S. natural gas distribution network in 2007. We then compare the robust energy flows with the transmission capacity (*TC*) of a corresponding pipeline. We use the ratio of robust energy flows to transmission capacity (i.e., *RCF/TC*) to indicate how congested a pipeline may be. If the ratio is greater than 1, the pipeline will be identified as a bottleneck.

Based on the macroscopic evaluation of the robustness of a distribution network, a natural way to calculate the robust energy flows is to maximize network entropy under a set of distribution

[1] The geographical information on the U.S. states is based on the regional divisions used by the United States Census Bureau, which consists of four regions and nine divisions.

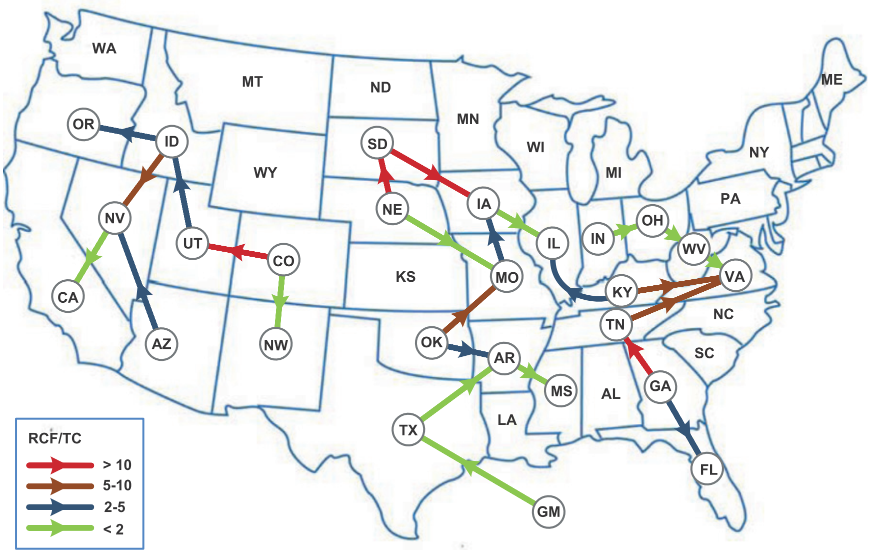

Fig. 3. The identified bottlenecks in the U.S. natural gas distribution network in 2007. The color of the arrows implies the ratio of *RCF* to *TC*. The larger the ratio, the more congested the interstate connection may be. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

constraints; that is, to solve the following constrained optimization problem:

$$\text{maximize} \quad H(P,\Pi^{-s}|\Pi^{s}) = \sum_{v_i \notin S} H_i \tag{7}$$

$$\text{subject to} \quad \sum_{e_{ji}=1} \pi_j p_{ji} = \sum_{e_{ik}=1} \pi_i p_{ik}, \quad \forall v_i, v_j \in V \tag{8}$$

$$\sum_{e_{ij}=1} p_{ij} = 1, \quad \forall v_i \in V \tag{9}$$

$$p_{ij} \geq 0, \quad \forall v_i, v_j \in V \tag{10}$$

where $H_i = -\pi_i \cdot \sum_{e_{ij}=1} p_{ij} \log p_{ij}$. According to Eq. (8), the inflows of a node should be equal to its outflows; that is, it should follow the property of flow conservation. Eq. (9) shows that a node will always forward all received inflows to its downstream neighbors. Eq. (10) avoids negative flows on each edge. Given the supply of each supplier and the demand of each consumer, an optimal solution exists because the objective function is continuous and the feasible region is compact.

In this case study, we focus on identifying the bottlenecks of the interstate natural gas pipeline network within the U.S. from the perspective of distribution robustness. We initialize the interstate connections and the supply/demand of each state based on the U.S. natural gas statistics in the year 2007. To solve the constrained nonlinear optimization problem (i.e., Eqs. (7)–(10)), we use Matlab's *fmincon* function with default settings. The solution of the optimization problem yields the optimal quantity of natural gas flows between the U.S. states. Therefore, for each interstate connection we can examine whether its transmission capacity can satisfy the potential utilization for robust distribution. The bottlenecks identified based on the optimization method are depicted in Fig. 3, where the color of each arrow implies the ratio of *RCF* to *TC*.

Table 2 provides details of the 25 bottlenecks identified in the 161 interstate pipeline connections in terms of their distribution robustness. The first column shows the interstate connections that are identified as bottlenecks. The second column shows the optimal flows of corresponding interstate connections. The third column shows the integrated real-world transmission capacity between the two U.S. states. The fourth column shows the ratio of *RCF* to *TC*. From the perspective of distribution robustness, the larger the ratio, the more congested the interstate connection may be. It can be seen that for certain interstate connections, the value of *RCF* is much larger than *TC*. This is because the *RCF* in this work is calculated by considering only distribution robustness. However, the real-world distribution of natural gas and the construction of pipelines may be affected by multiple factors, such as transmission costs, pipeline investment capital, geographical conditions, and socioeconomic factors. Nevertheless, the results presented here still offer a new perspective on the capacity management of the U.S. natural gas pipeline network. In the future, it would be interesting to extend the optimization problem for identifying transmission congestion by considering both distribution robustness and transmission costs. This can be done

Table 2
The 25 bottlenecks identified out of the 161 U.S. interstate connections in terms of their distribution robustness.

Interstate Connection	RCF (10^5 *MMcf*)	TC (10^5 *MMcf*)	Ratio (*RCF*/*TC*)
SD→IA	5.652	0.073	77.421
GA→TN	11.931	0.314	38.010
NE→SD	3.079	0.146	21.214
CO→UT	7.663	0.763	10.045
OK→MO	2.747	0.292	9.409
ID→NV	5.283	0.577	9.161
KY→VA	1.302	0.183	7.134
TN→VA	1.302	0.256	5.096
UT→ID	10.200	2.223	4.589
AZ→NV	4.112	1.073	3.832
ID→OR	5.506	1.566	3.516
GA→FL	4.575	1.500	3.050
MO→IA	7.016	2.482	2.827
KY→IL	12.790	5.636	2.270
OK→AR	16.710	7.399	2.258
NV→CA	10.170	6.490	1.567
AR→MS	36.390	25.080	1.451
GM→TX	15.710	11.580	1.357
TX→AR	10.190	7.563	1.347
IN→OH	25.200	19.330	1.304
NE→MO	3.162	2.529	1.250
OH→WV	12.250	10.060	1.219
CO→NW	9.738	8.019	1.214
WV→VA	9.045	7.869	1.149
IA→IL	24.040	21.690	1.108

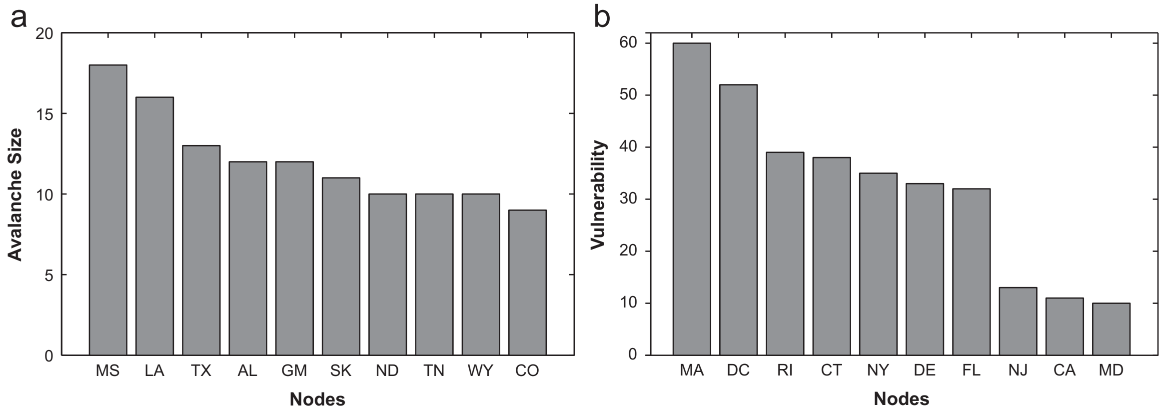

Fig. 4. Top 10 most important and vulnerable U.S. states in terms of natural gas distribution. The left figure shows the top 10 most important U.S. states, and the right figure shows the top 10 most vulnerable U.S. states.

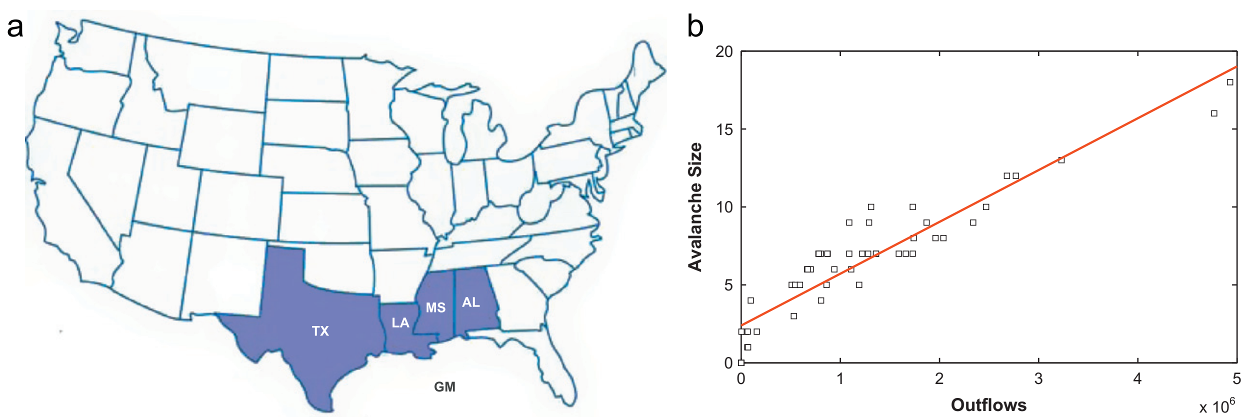

Fig. 5. The important regions and the relationship between the outflows of nodes and their avalanche size in the U.S. natural gas distribution network in 2007. The left figure (a) shows that the top five most important nodes are located around the Gulf of Mexico. The right figure (b) shows that the nodes' outflows have a significant correlation with their avalanche sizes.

by integrating our model with the concept of locational marginal price, which was introduced by Schweppe et al. (1988), and further developed and adopted by Hogan (1992), Quelhas et al. (2007), Lochner (2011), and so forth.

4.3. Ranking the importance and vulnerability of the nodes

The importance of a node is measured by the number of failures it can cause (the avalanche size), whereas the vulnerability of a node is measured by the size of its dominant set. Fig. 4 shows the top 10 most important and most vulnerable nodes in the U.S. natural gas distribution network in 2007 with respect to avalanche size (Fig. 4(a)) and vulnerability (Fig. 4(b)). It can be observed that the top five most influential nodes are located around the southern region near the Gulf of Mexico (i.e., Mississippi (MS), Louisiana (LA), Texas (TX), Alabama (AL), and the Gulf of Mexico (GM), as shown in Fig. 5(a)), which are the major natural gas production areas. Therefore, one would expect the production of a node would determine the avalanche size of its dominant set. However, this is not the case. We find that it is the nodes' outflows that have a significant (p-value <0.05) correlation with their avalanche sizes (Fig. 5(b)). Intuitively, one would also expect that a node's vulnerability would be related to its consumption or inflows. However, we cannot find any significant relationship between vulnerability and these two factors. In fact, existing reports have shown that a node's vulnerability may be affected by a combination of many factors, such as its consumption and its position in the network, as well as network connectivity and flow dynamics on the distribution network.

To evaluate the failure spreading model, we compare it with the eigenvector-based approach (EIG) using an "extinction" procedure. The EIG approach was proposed by Allesina and Pascual (2009) to measure species' importance for coextinctions in food webs and has been validated with a better performance than other existing approaches, including degree centrality, closeness centrality, betweenness centrality, and dominators (Allesina and Bodini, 2004). The extinction procedure is as follows: we disrupt the nodes on the U.S. distribution network one by one, following the importance sequences generated by different approaches. After each disruption, we record the proportion of failures. For each approach, we measure the "failure area", a quantity similar to the "extinction area" proposed by Allesina and Pascual (2009), where we calculate the proportion of failed nodes after the nodes are sequentially disrupted. The area equals 1 when all nodes fail after the first node is disrupted and tends to be 1/2 when no secondary failure occurs. We can therefore assess the performance of each approach according to the value of the failure area. If important nodes are disrupted early on, then the value of the failure area tends to be larger. Fig. 6 shows the failure areas of the eigenvector-based approach (EIG) and the failure spreading model (FSM) in terms of the U.S. natural gas distribution network in 2007. The failure area is described by the area below the curves. The results show that the FSM approach is more effective in identifying the node importance than the EIG approach. Similar

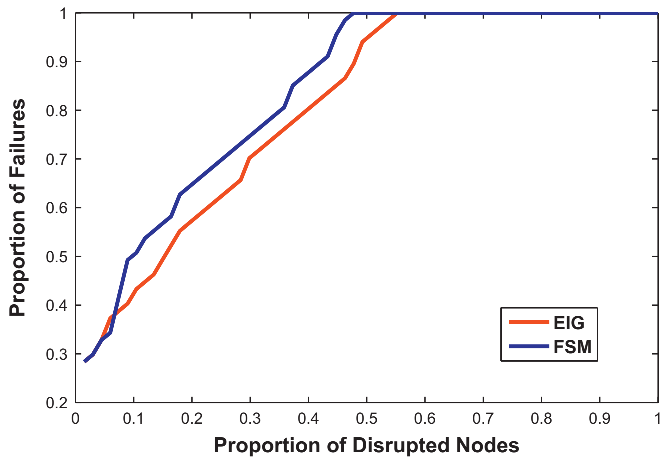

Fig. 6. The failure areas are described by the areas below the curves. Each failure area can take a value from 1/2 to 1. The blue and red curves are generated using the failure spreading model (FSM) and the eigenvector-based approach (EIG) in terms of the U.S. natural gas distribution network in 2007. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

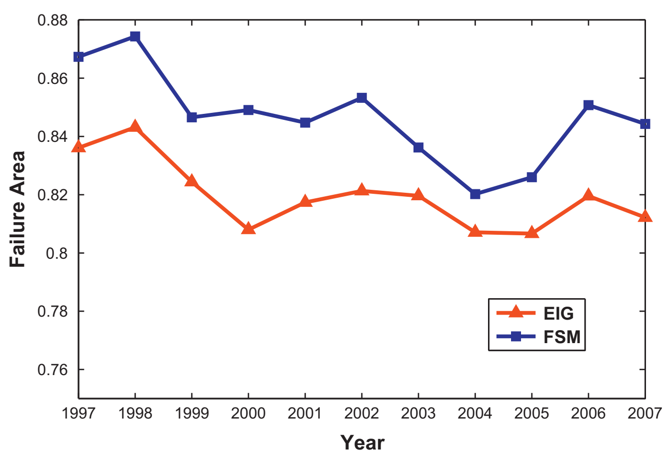

Fig. 7. The values of the failure areas with respect to the U.S. natural gas distribution network from 1997 to 2007. The blue and red curves represent the values calculated using the FSM and EIG approaches. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

comparisons are made between the EIG and FSM approaches by calculating their failure areas in terms of the U.S. natural gas distribution network from 1997 to 2007. Fig. 7 shows the values of the failure area with respect to different years. We can see that the FSM approach always performs better than the EIG approach.

4.4. Identifying advantageous storage locations

In reality, the storage of natural gas, which is accessed directly by local distribution companies, plays a key role in alleviating demand fluctuations, especially the demand changes from the non-heating season to the heating season. Moreover, storage allows the distribution system to respond quickly to sudden shifts in demand (EIA, 1995). However, from a systematic perspective, allowing nodes to manage their storage individually may not be an efficient way to mitigate disruptions. There should be some storage locations that are geographically convenient to mitigate the impacts of supply disruptions on other nodes. Here, we aim to identify such nodes or locations using a hierarchical structure (Fig. 8), which represents the relationships between the dominant sets of the nodes.

Several observations can be made from Fig. 8. First, based on the formation rule of the hierarchical structure, a node in a path is more vulnerable than its downstream nodes. Therefore, we find that the northeast region is more vulnerable than other regions. This can be verified by the existing reports because the demand in the region relies heavily on the supply from the Gulf Coast, Rocky Mountain, or Canada (EIA, 2007a,c). Second, the regional branch points, such as Massachusetts (MA), California (CA), and Delaware (DE), may be significant in terms of mitigating disruptions. There are two reasons: (i) a branch point is more vulnerable than its downstream nodes and (ii) a branch point may imply that it has multiple supply alternatives. When a disruption occurs on one route, the branch node may receive energy from alternative sources to mitigate the disruption. For example, Massachusetts receives natural gas from Canada (through ME and NH), the Gulf Coast, and the West, and California receives natural gas from both Canada and New Mexico (through Arizona). Moreover, if other nodes are geographically close to and connected to the node, this can considerably alleviate the disruption. During the El Paso pipeline disruption in 2000, California benefitted from its supply diversity and New Mexico benefitted from its extensive production. Arizona inversely benefitted from California's storage if California's pipelines drew natural gas from their storage facilities while permitting the El Paso and Transwestern Pipeline systems to divert supplies normally directed to California (EIA, 2000). Third, different regional distribution patterns can be observed from Fig. 8. For the Northeast region, it is clear that there are two groups of nodes, with RI, CT, NY, NJ, and PA as one group, which relies on domestic supply, and NH and ME as another, which relies on supply from the east of Canada. For the Midwest region, MI, WI, SD, and ND form a group that relies on supply from Canada. For the South region, one obvious group is TN, GA, SC, and NC, which heavily depends on supply from the Gulf Coast. For the West region, WA and OR rely on supply from the West of Canada, whereas AZ and NV may receive supply from both New Mexico and Rocky Mountain.

4.5. Discussion on mitigation strategies

From our findings, we have seen several implications for mitigating the impacts of supply disruptions. First, the failure spreading model in this paper can efficiently identify those important nodes whose failure may extensively disrupt other nodes. In doing so, a natural way of improving the robustness of a distribution network is to pay more attention to protecting identified important nodes from being disrupted. This can be achieved by integrating energy accidents into the design of the energy infrastructure, improving the prediction accuracy of recurrent accidents, establishing response guidelines for emergencies, and so forth. Second, the proposed hierarchical structure based on the results of the failure spreading model can help identify the advantageous nodes in the U.S. distribution network for locating the energy storage. Based on the analysis, the identified vital nodes have two properties: (i) they are more vulnerable than other nodes in the same region and (ii) their storages are convenient for mitigating disruption impacts on other nodes. Therefore, the storage locations of energy resources should be planned in a systematic manner. Third, and most importantly, the network entropy, which is defined in terms of the dynamics of energy flows on a distribution network, can be used to evaluate the robustness of distribution network. Consequently, we can calculate the optimal energy flows on the network by solving a network optimization problem under some distribution constraints. In this paper, by comparing the

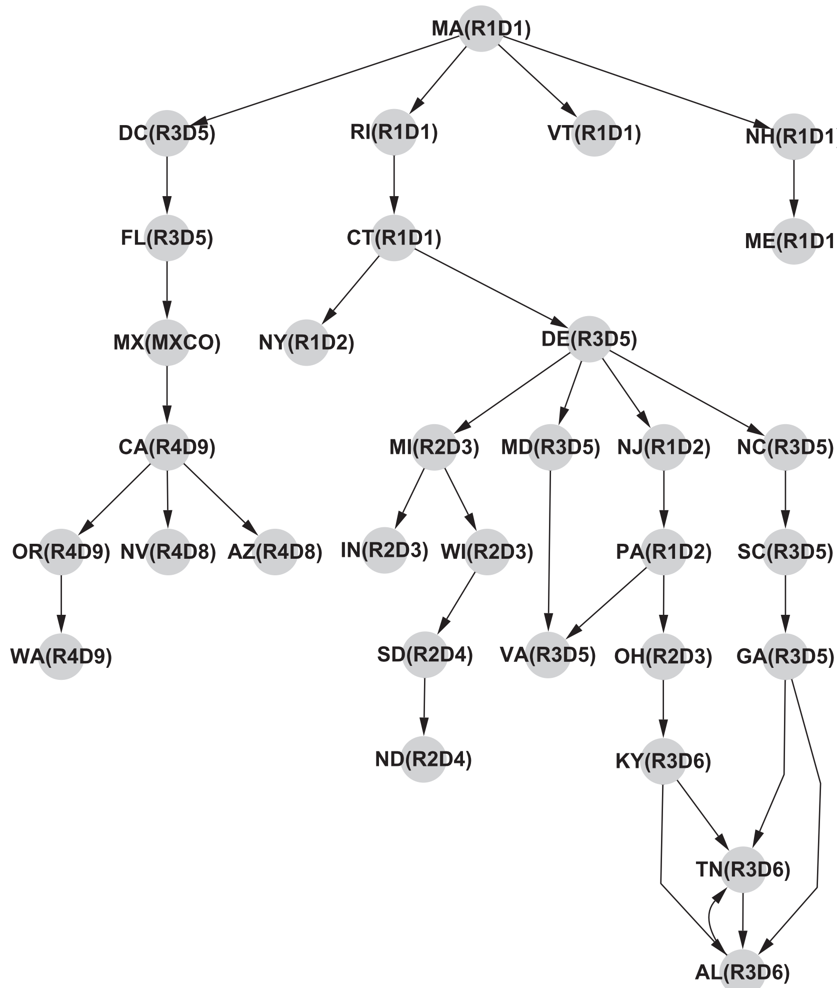

Fig. 8. The hierarchical structure of the dominant sets of the U.S. natural gas transmission network in 2007. The geographical information is associated with each node n parentheses.

calculated flows with real-world transmission capacity of each transmission connection, we can identify the transmission bottlenecks of a distribution network, whose transmission capacity should be improved to satisfy robust distribution. This provides a new perspective for the construction of transmission infrastructure at the systems level. Furthermore, such a definition is also helpful for designing market-oriented mechanisms or policies to guide the decision-making of energy suppliers and consumers so as to enhance the distribution robustness. According to the complex systems approach proposed by Beck et al. (2008), participants in such mechanisms should be capable of making their own decisions for planning and multi-objective optimization following the proposed rules or policies.

5. Conclusion

To quantitatively evaluate and hence better understand the robustness of an energy distribution network in the face of supply disruptions, in this paper, we have focused on two levels of investigation: (i) how to macroscopically characterize distribution robustness with respect to the dynamics of energy flows on the network and (ii) how to microscopically evaluate the extent to which supply disruptions on one node may affect the others. First, we formally defined a notion of network entropy based on the Ruelle–Bowens random walk theory from the perspective of complex networks. We showed how such a macroscopic evaluation leads to a new way of enhancing distribution robustness, by solving a network entropy optimization problem under certain distribution constraints. By doing so, we can further evaluate the bottlenecks of distribution infrastructures by comparing calculated robust energy flows with corresponding distribution capacities. Next, we adopted a failure spreading model to evaluate the robustness of a distribution network at the microscopic level. Such a microscopic insight can readily help us rank the disruption impacts of individual nodes as well as determine the interdependency among them in the face of supply disruptions. Furthermore, we demonstrated our proposed concepts and methods by using the publically available statistical data on the United States natural gas distribution network. Specifically, we showed how the distribution bottlenecks of the U.S. interstate pipeline connections can be identified based on our macroscopic evaluation. Then, using the microscopic failure spreading model, we ranked the nodes' importance (measured by the number of failures it may incur) and vulnerability (measured by the number of nodes that may cause its failure), and identified advantageous natural gas storage locations that are geographically convenient for mitigating the disruptions impacts on their neighbor states. Finally, we discussed several implications of our findings for mitigating the impacts of supply disruptions.

It should be pointed out that given the aggregated nature of the publically available statistical data, the U.S. natural gas distribution network that we have examined in this paper is of a relatively small size. Nevertheless, the methods introduced here can readily be applied to evaluate larger-scale (finer-grained) energy distribution networks. As for our future work, we will consider extensions along the following directions. First of all, in our failure spreading model, we assume that all nodes have the same possibility to be disrupted. However, similar to natural hazards (Cutter and Finch, 2008), real-world supply disruptions on a specific energy distribution network may have some temporal and spatial patterns. Thus, to more holistically understand the robustness of a specific distribution network, it would be desirable to analyze historical disruptions on the network so that the disruption possibility can be statistically estimated. Second, as introduced by Beck et al. (2008), entities in an energy distribution network such as suppliers and consumers are by nature heterogeneous and interactive, who autonomously make their decisions. In this regard, it would be interesting to see how our findings can help policy makers develop new policies to motivate such entities to robustly distribute energy in the absence of a centralized control. Finally, we will further extend the macroscopic and microscopic evaluations to integrating the issue of distribution robustness with other issues such as transmission costs, so as to distribute energy resources more reliably, efficiently, and economically among energy suppliers and consumers.

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References

Albert, R., Albert, I., Nakarado, G.L., 2004. Structural vulnerability of the North American power grid. Physical Review E 69 (2), 025103.
Albert, R., Barabási, A.L., 2002. Statistical mechanics of complex networks. Reviews of Modern Physics 74 (1), 47–97.
Allesina, S., Bodini, A., 2004. Who dominates whom in the ecosystem? Energy flow bottlenecks and cascading extinctions. Journal of Theoretical Biology 230 (3), 351–358.
Allesina, S., Pascual, M., 2009. Googling food webs: can an eigenvector measure species' importance for coextinctions? PLoS Computational Biology 5 (9), e1000494.
Anand, K., Bianconi, G., 2009. Entropy measures for complex networks: toward an information theory of complex topologies. Physical Review E 80 (4), 045102.
Beck, J., Kempener, R., Cohen, B., Petrie, J., 2008. A complex systems approach to planning, optimization and decision making for energy networks. Energy Policy 36 (8), 2795–2805.
Blöchl, F., Theis, F.J., Vega-Redondo, F., Fisher, E.O., 2011. Vertex centralities in input-output networks reveal the structure of modern economies. Physical Review E 83 (4), 046127.
Boccaletti, S., Latora, V., Moreno, Y., Chavezf, M., Hwang, D., 2006. Complex networks: structure and dynamics. Physics Reports 424 (4–5), 175–308.
Brin, S., Page, L., 1998. The anatomy of a large-scale hypertextual web search engine. Computer Networks and ISDN Systems 30, 107–117.
Burgherr, P., Hirschberg, S., 2008. Severe accident risks in fossil energy chains: a comparative analysis. Energy 33 (4), 538–553.
Cutter, S.L., Finch, C., 2008. Temporal and spatial changes in social vulnerability to natural hazards. Proceedings of the National Academy of Sciences of the United States of America 105 (7), 2301–2306.
Delvenne, J.C., Libert, A.S., 2011. Centrality measures and thermodynamic formalism for complex networks. Physical Review E 83 (4), 046117.
Demetrius, L., Manke, T., 2005. Robustness and network evolution: an entropic principle. Physica A: Statistical Mechanics and its Applications 346 (3–4), 682–696.
EIA, 1995. The Value of Underground Storage in Today's Natural Gas Industry. Technical Report. U.S. Energy Information Administration.
EIA, 2000. A Look at Western Natural Gas Infrastructure During the Recent El Paso Pipeline Disruption. Technical Report. U.S. Energy Information Administration.
EIA, 2007a. Annual Report of Natural and Supplemental Gas Supply and Disposition. Technical Report. U.S. Energy Information Administration.
EIA, 2007b. International and Interstate Movements of Natural Gas by State. U.S. Energy Information Administration 〈http://www.eia.gov/dnav/ng/ng_move_ist_a2dcu_nus_a.htm〉 (accessed April 2010).
EIA, 2007c. Natural Gas Annual 2007. Technical Report. U.S. Energy Information Administration.
EIA, 2007d. Natural Gas Annual Supply and Disposition by State. U.S. Energy Information Administration 〈http://www.eia.gov/dnav/ng/ng_sum_snd_dcu_NUS_a.htm〉 (accessed April 2010).
EIA, 2007e. State to State Natural Gas Pipeline Capacities and Flows. U.S. Energy Information Administration 〈http://www.eia.gov/pub/oil_gas/natural_gas/analysis_publications/ngpipeline/usage.html〉 (accessed April 2010).
Federal Energy Regulatory Commission, 2006. 2006 State of the Markets Report. Technical Report. Federal Energy Regulatory Commission.
Garas, A., Argyrakis, P., Rozenblat, C., Tomassini, M., Havlin, S., 2010. Worldwide spreading of economic crisis. New Journal of Physics 12 (11), 113043.
Gómez-Gardeñes, J., Latora, V., 2008. Entropy rate of diffusion processes on complex networks. Physical Review E 78 (6), 065102.
Haldane, A.G., May, R.M., 2011. Systemic risk in banking ecosystems. Nature 469, 251–355.
Hogan, W.W., 1992. Contract networks for electric power transmission. Journal of Regulatory Economics 4 (3), 211–242.
Jamasb, T., Pollitt, M., 2008. Security of supply and regulation of energy networks. Energy Policy 36 (8), 4584–4586.
Kabirian, A., Hemmati, M.R., 2007. A strategic planning model for natural gas transmission networks. Energy Policy 35 (11), 5656–5670.
Kinney, R., Crucitti, P., Albert, R., Latora, V., 2005. Modeling cascading failures in the North American power grid. The European Physical Journal B – Condensed Matter and Complex Systems 64 (1), 101–107.
Kumar, A., Faltings, B., Petcu, A., 2009. Distributed constraint optimization with structured resource constraints. In: Proceedings of the 8th International Conference on Autonomous Agents and Multiagent Systems, pp. 923–930.
Lee, K.M., Yang, J.S., Kim, G., Lee, J., Goh, K.I., Kim, I.M., 2011. Impact of the topology of global macroeconomic network on the spreading of economic crises. PLoS One 6 (3), e18443.
Lochner, S., 2011. Identification of congestion and valuation of transport infrastructures in the European natural gas market. Energy 36 (5), 2483–2492.
May, R.M., Arinaminpathy, N., 2010. Systemic risk: the dynamics of model banking systems. Journal of the Royal Society Interface 7, 823–838.
Motter, A.E., Lai, Y.C., 2002. Cascade-based attacks on complex networks. Physical Review E 66 (6), 065102.
Newman, M.E.J., 2003. The structure and function of complex networks. SIAM Review 45 (2), 167–256.
Quelhas, A., Gil, E., McCalley, J.D., Ryan, S.M., 2007. A multiperiod generalized network flow model of the U.S. integrated energy system: part I – model description. IEEE Transaction on Power Systems 22 (2), 829–836.
Quelhas, A., McCalley, J.D., 2007. A multiperiod generalized network flow model of the U.S. integrated energy system: part II – simulation results. IEEE Transaction on Power Systems 22 (2), 837–844.
Scholtens, B., Boersen, A., 2011. Stocks and energy shocks: the impact of energy accidents on stock market value. Energy 36 (3), 1698–1702.
Schweppe, F., Caramanis, M., Tabors, R., Bohn, R., 1988. Spot Pricing of Electricity. Kluwer Academic Publishers.
Shakkottai, S., Srikant, R., 2007. Network optimization and control. Foundations and Trends in Networking 2 (3), 271–379.
Shaw, R., Attree, M., Jackson, T., 2010. Developing electricity distribution networks and their regulation to support sustainable energy. Energy Policy 38 (10), 5927–5937.
Shaw, R., Attree, M., Jackson, T., Kay, M., 2009. The value of reducing distribution losses by domestic load-shifting: a network perspective. Energy Policy 37 (8), 3159–3167.
Solé, R.V., Valverde, S., 2004. Information theory of complex networks: on evolution and architectural constraints. In: Lecture Notes in Physics, vol. 650, pp. 189–207.
Sovacool, B.K., 2008. The costs of failure: a preliminary assessment of major energy accidents, 1907-2007. Energy Policy 36 (5), 1802–1820.
van der Vleuten, E., Lagendijk, V., 2010a. Interpreting transnational infrastructure vulnerability: European blackout and the historical dynamics of transnational electricity governance. Energy Policy 38 (4), 2053–2062.
van der Vleuten, E., Lagendijk, V., 2010b. Transnational infrastructure vulnerability: the historical shaping of the 2006 European "Blackout". Energy Policy 38 (4), 2042–2052.
Wang, B., Tang, H., Guo, C., Xiu, Z., 2006. Entropy optimization of scale-free networks robustness to random failures. Physica A 363 (2), 591–596.